

Calculation of Boson Masses in Higher New Tamm–Dancoff-Approximation in the Nonlinear Spinor Theory

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Eigenvalue equations of bosons in the second step of the New TAMM-DANCOFF-approximation (NTD) are investigated in the HERMITEAN formulation of the nonlinear spinor theory.

It is shown that three of the four derived equations lead to the same result, whereas one approximation is completely different and is supposed to give a useless result, because the suppressed 6-point function contains essential parts of the considered 2-point function. It is possible by a factorization of the equations of the second iteration to calculate a local graph. By taking this graph into consideration the eigenvalues of spin-0-particles obtained in lowest NTD are slightly corrected. For comparison the same questions are discussed in case of the anharmonic oscillator.

§ 1. Nonlinear Spinor Theory with Hermitean Field Operators¹

We start with the nonlinear spinor equation in the Hermitean form

$$D_{\alpha\beta}\psi_\beta = V_{\alpha\beta\gamma\delta}:\psi_\beta\psi_\gamma\psi_\delta: \quad (1.1)$$

with

$$D_{\alpha\beta} = -i(\lambda'_2\sigma'^r)_{\alpha\beta}\delta(x_\alpha - x_\beta)\frac{\partial}{\partial x_\beta^r}, \quad (1.2)$$

$$V_{\alpha\beta\gamma\delta} = \frac{1}{2}l^2\delta(x_\beta - x_\alpha)\delta(x_\gamma - x_\alpha)\delta(x_\delta - x_\alpha)\bar{V}_{\alpha\beta\gamma\delta}, \quad (1.3)$$

$$\bar{V}_{\alpha\beta\gamma\delta} = \frac{1}{6}[3\sigma'^\mu\sigma'_\mu + \lambda'_2\lambda'_2\tau'\tau'\sigma'^\mu\sigma'_\mu + 2(\lambda'_1\lambda'_1 + \lambda'_3\lambda'_3)\tau'\tau'c_\sigma c_\sigma]_{\alpha\beta\gamma\delta}. \quad (1.4)$$

(For the definition of the λ' , σ' , τ' -matrices and their commutation rules see¹.) For the “normal-ordered product” of the field operators (indicated by : :) we have with the time-ordering operator T

$$V_{\alpha\beta\gamma\delta}:\psi_\beta\psi_\gamma\psi_\delta: = V_{\alpha\beta\gamma\delta}[T\psi_\beta\psi_\gamma\psi_\delta - 3F_{\beta\gamma}\psi_\delta] \quad (1.5)$$

with the contraction function

$$F_{\beta\gamma} = \langle 0|T\psi_\beta\psi_\gamma|0\rangle. \quad (1.6)$$

Defining the τ -functions

$$\tau_{AB}^{(k)}(1\dots k) = \langle A|T\psi_1\dots\psi_k|B\rangle \quad (1.7)$$

from (1.1) we get for the functional generating the τ -functions

$$\tau_{AB}(u) = \langle A|T\exp iu_\alpha\psi_\alpha|B\rangle \quad (1.8)$$

$$= \sum_k \frac{i^k}{k!} u_k\dots u_1 \tau_{AB}^{(k)}(1\dots k)$$

the functional equation

$$\left\{ D_{\alpha\beta} \frac{\delta}{\delta u_\beta} + V_{\alpha\beta\gamma\delta} \left[\frac{\delta^3}{\delta u_\beta \delta u_\gamma \delta u_\delta} + 3F_{\beta\gamma} \frac{\delta}{\delta u_\delta} \right] \right\} \tau_{AB}(u) = 0. \quad (1.9)$$

Here u is a real spinor source function anticommuting with itself and the field operator ψ .

To solve the equations we transform the τ -functions into the φ -functions using the functional connection

$$\Phi_{AB}(u) = e^{-\frac{1}{2}u_1 F_{12} u_2} \tau_{AB}(u) \quad (1.10)$$

and

$$\Phi_{AB}(u) = \sum_k \frac{i^k}{k!} u_k\dots u_1 \varphi_{AB}^{(k)}(1\dots k). \quad (1.11)$$

We then find the functional equation

$$\left\{ D_{\alpha\beta} \frac{\delta}{\delta u_\beta} + V_{\alpha\beta\gamma\delta} []_{\beta\gamma\delta} + D_{\alpha\beta} F_{\beta\gamma} u_\gamma \right\} \Phi_{AB}(u) = 0 \quad (1.12)$$

or using the GREEN-Function of (1.1) $G_{\alpha\beta}$

$$\left\{ \frac{\delta}{\delta u_0} + G_{0\alpha} V_{\alpha\beta\gamma\delta} []_{\beta\gamma\delta} + F_{0\alpha} u_\alpha \right\} \Phi_{AB}(u) = 0 \quad (1.13)$$

¹ H. P. DÜRR and F. WAGNER, Nuovo Cimento 46, 223 [1966].



with

$$[\]_{\beta\gamma\delta} = \frac{\delta^3}{\delta u_\beta \delta u_\gamma \delta u_\delta} + 3 F_{\delta l} u_l \frac{\delta^2}{\delta u_\beta \delta u_\gamma} + 3 F_{\beta i} F_{\gamma j} u_i u_j \frac{\delta}{\delta u_\delta} + F_{\beta i} F_{\gamma j} F_{\delta l} u_i u_j u_l. \quad (1.14)$$

We can collect several expressions in (1.13) and get

$$\begin{aligned} & \left(\delta_{0\gamma} + 3 G_{0\alpha} V_{\alpha\beta\gamma\delta} F_{\delta l} u_l \frac{\delta}{\delta u_\beta} \right) \left(\frac{\delta}{\delta u_\gamma} + F_{\gamma i} u_i \right) \Phi_{AB}(u) \\ &= -G_{0\alpha} V_{\alpha\beta\gamma\delta} \left(\frac{\delta^3}{\delta u_\beta \delta u_\gamma \delta u_\delta} + F_{\beta i} F_{\gamma j} F_{\delta l} u_i u_j u_l \right) \Phi_{AB}(u). \end{aligned} \quad (1.15)$$

This form is used for further investigations and allows us to solve the eigenvalue equations in the second NTD in a relatively simple way.

From (1.13) we get the following system of integral equations

$$\varphi^{(k+1)}(01 \dots k) = G_{0\alpha} V_{\alpha\beta\gamma\delta} [(\beta \gamma \delta 1 \dots k)] - \sum_j (-)^{j+1} \varphi^{(k-1)}(1 \dots, j-1, j+1, \dots k) F_{0j} \quad (1.16)$$

where

$$\begin{aligned} [(\beta \gamma \delta 1 \dots k)] &= \varphi^{(k+3)}(\beta \gamma \delta 1 \dots k) + \sum_{(1)} \varphi_F^{(k+1)}(\beta \gamma \delta 1 \dots k) \\ &+ \sum_{(2)} \varphi_{FF}^{(k-1)}(\beta \gamma \delta 1 \dots k) + \sum_{(3)} \varphi_{FFF}^{(k-3)}(\beta \gamma \delta 1 \dots k); \end{aligned} \quad (1.17)$$

$\sum_{(1)} \varphi_F$, $\sum_{(2)} \varphi_{FF}$, $\sum_{(3)} \varphi_{FFF}$ indicate sums over φ -functions with all single, double and triple contractions, i.e.

$$\sum_{(1)} \varphi_F^{(k+1)} = 3 \sum_l (-)^{l+1} F_{\delta l} \varphi^{(k+1)}(\beta \gamma \dots, l-1, l+1, \dots), \quad (1.18a)$$

$$\sum_{(2)} \varphi_{FF}^{(k-1)} = 6 \sum_{l < j} (-)^{l+j+1} F_{\delta l} F_{\gamma j} \varphi^{(k-1)}(\beta \dots, l-1, l+1, \dots, j-1, j+1, \dots), \quad (1.18b)$$

$$\sum_{(3)} \varphi_{FFF}^{(k-3)} = 6 \sum_{l < j < i} (-)^{l+j+i} F_{\delta l} F_{\gamma j} F_{\beta i} \varphi^{(k-3)}(\dots, l-1, l+1, \dots, j-1, j+1, \dots, i-1, i+1, \dots). \quad (1.18c)$$

The considerations are facilitated by using the following graphical representation¹

$$\begin{aligned} G_{12} &= \text{graph} = \text{graph} = - \text{graph}, & V_{\alpha\beta\gamma\delta} &= \text{graph}, \\ F_{12} &= \text{graph} = \text{graph} = - \text{graph}, & \varphi_{0B}(1 \dots k) &= \text{graph}. \end{aligned} \quad (1.19)$$

As vacuum 2-point function we assume the form used in earlier papers². So we arrive at the following meaning of the graphs in momentum space:

$$\begin{aligned} \text{---}_p &= G(p) = -\lambda_2' \bar{\sigma}_\mu' \frac{p^\mu}{p^2 + i\epsilon} \\ \text{---}_p^\circ &= F(p) = -i\lambda_2' \bar{\sigma}_\mu' \frac{p^\mu \kappa^4}{(p^2 - \kappa^2 + i\epsilon)(p^2 + i\epsilon)^2} \\ \text{---} \cdot \text{---} &= \frac{1}{2} l^2 \bar{V} = \frac{1}{2} l^2 V_i V_i \\ V_i V_i &= \frac{1}{6} (3 + \lambda_2' \lambda_2' \tau' \tau') \sigma' \mu \sigma'_\mu + \frac{1}{3} (\lambda_1'' \lambda_1'' + \lambda_3'' \lambda_3'') \tau' \tau'. \end{aligned} \quad (1.20)$$

§ 2. Boson Eigenvalue Equations

The boson matrix elements $\langle 0 | \dots | B \rangle$ fulfil the following integral equation, derived from Eq. (1.16) for the $2n$ - φ -functions:

$$\text{graph} = \text{graph} - 3 \text{graph} \quad (2.1)$$

² H. RAMPACHER, H. STUMPF, and F. WAGNER, Fortschr. Physik **13**, 385 [1965].

an analogous equation, where the lower leg is iterated, and

$$\begin{aligned} \text{Diagram 1} &= \text{Diagram 2} - 3 \left[\text{Diagram 3} - \text{Diagram 4} + \text{Diagram 5} \right] \text{Diagram 6} \\ &+ 6 \left[\text{Diagram 7} - \text{Diagram 8} + \text{Diagram 9} \right] \text{Diagram 10} - \left[\text{Diagram 11} - \text{Diagram 12} + \text{Diagram 13} \right] \text{Diagram 14}, \end{aligned} \quad (2.2a)$$

$$\begin{aligned} \text{Diagram 1} &= \text{Diagram 2} - 3 \left[\text{Diagram 3} - \text{Diagram 4} + \text{Diagram 5} \right] \text{Diagram 6} \\ &+ 6 \left[\text{Diagram 7} - \text{Diagram 8} + \text{Diagram 9} \right] \text{Diagram 10} - \left[\text{Diagram 11} - \text{Diagram 12} + \text{Diagram 13} \right] \text{Diagram 14}. \end{aligned} \quad (2.2b)$$

We have explicitly written down the equation, derived by iterating the fourth leg, because we shall get from it a different result than by iterating one of the first three legs.

With the aid of (1.15) (2.2) can be cast into the form

$$\left(1 + 3 \left[\text{Diagram 3} - \text{Diagram 4} + \text{Diagram 5} \right] \right) \left(\text{Diagram 6} + \text{Diagram 11} - \text{Diagram 12} + \text{Diagram 13} \right) = \text{Diagram 1}, \quad (2.3a)$$

$$\left(1 + 3 \left[\text{Diagram 7} - \text{Diagram 8} + \text{Diagram 9} \right] \right) \left(\text{Diagram 6} + \text{Diagram 11} - \text{Diagram 12} + \text{Diagram 13} \right) = \text{Diagram 1}. \quad (2.3b)$$

If we invert the operator on the left hand side without respect to questions of convergence, we get respectively

$$\text{Diagram 1} = - \text{Diagram 11} + \text{Diagram 12} - \text{Diagram 13} + \text{Diagram 6} + \varphi_a^{(6)}, \quad (2.4a)$$

$$\text{Diagram 1} = - \text{Diagram 11} + \text{Diagram 12} - \text{Diagram 13} + \text{Diagram 6} + \varphi_b^{(6)}, \quad (2.4b)$$

where in the $\varphi_i^{(6)}$'s consist of 6-point functions with at least one contraction function. By inserting we recognize that these expressions fulfil the Eqs. (2.2) apart from 6-point functions.

With (2.4) we get from (2.1) new equations for the 2-point function, namely from (2.2a)

$$\text{Diagram 1} = - 2 \text{Diagram 3} + \bar{\varphi}_a^{(6)} + \text{Diagram 6} \quad (2.5)$$

and from (2.2b)

$$\text{Diagram 1} = \text{Diagram 6} + \bar{\varphi}_b^{(6)}, \quad (2.6)$$

where the $\bar{\varphi}_i^{(6)}$'s mean appropriately changed sums of 6-point functions.

Neglecting the 6-point functions we derive from (2.5) an equation slightly changed compared to (2.1) in the first NTD (the factor 3 is replaced by 2):

$$\text{Diagram 1} = - 2 \text{Diagram 3}. \quad (2.7a)$$

From (2.6) follows that the 2-point function vanishes identically what is useless.

$$\text{Diagram 1} = 0. \quad (2.7b)$$

The situation here resembles that in the case of the twofold iteration of the fermion propagator in¹. There we also have four equations and by neglecting the symmetrically iterated η -6-point function we lose all important poles.

So we suppose that the following holds in general for boson calculations: If there appears in the equations for the $2n$ - q -functions a combination of graphs the lower and upper parts of which correspond to $-\square$ in the fermion calculation, then this combination contains essential parts of $-\square$ and cannot be neglected.

In addition it should be noted that we can use (1.15) in this simple way only in the second NTD. Writing down the equations for the 6-point function we have on the right hand side of (1.15) not only an 8-point function but also a 2-point function with three F -lines. But we suppose that (1.15) facilitates the solution also in this case.

The solution of (2.7) yields no new difficulties compared to¹. We get mass eigenvalue equations T_{SI}^B for baryon number B , spin S and isospin I

$$\begin{aligned} T_{00}^0 : q_0(x, \vartheta) &= -2 \left/ \left(\frac{\kappa l}{2\pi} \right)^2 \right., \\ T_{10}^0 : q_1(x, \vartheta) &= -2 \left/ \left(\frac{\kappa l}{2\pi} \right)^2 \right., \\ T_{01}^0 : q_0(x, \vartheta) &= -6 \left/ \left(\frac{\kappa l}{2\pi} \right)^2 \right., \\ T_{11}^0 : q_1(x, \vartheta) &= -6 \left/ \left(\frac{\kappa l}{2\pi} \right)^2 \right., \\ T_{01}^2 : q_0(x, \vartheta) + 3q_1(x, \vartheta) &= 6 \left/ \left(\frac{\kappa l}{2\pi} \right)^2 \right. \end{aligned} \quad (2.8)$$

Here u commutes with itself and with q . In this way we get the functional equation for $\tau_{AB}(u)$

$$\left[D_{\alpha\beta} \frac{\delta}{\delta u_\beta} + V_{\alpha\beta\gamma\delta} \frac{\delta^3}{\delta u_\beta \delta u_\gamma \delta u_\delta} - 3F_{\alpha\beta}(0) \frac{\delta}{\delta u_\beta} - q_0 i u_\alpha \right] \tau_{AB}(u) = 0 \quad (3.5)$$

with the abbreviations

$$D_{\alpha\beta} = \delta(t_\alpha - t_\beta) \frac{d^2}{dt_\beta^2} + 3F_{\alpha\beta}(0), \quad F_{\alpha\beta} = \langle 0 | T q(t_\alpha) q(t_\beta) | 0 \rangle, \quad (3.6), (3.7)$$

$$V_{\alpha\beta\gamma\delta} \frac{\delta^3}{\delta u_\beta \delta u_\gamma \delta u_\delta} = -\delta(t_\beta - t_\alpha) \delta(t_\gamma - t_\alpha) \delta(t_\delta - t_\alpha) \frac{\delta^3}{\delta u(t_\beta) \delta u(t_\gamma) \delta u(t_\delta)}. \quad (3.8)$$

In order to make the analogy to the spinor theory more manifest we have added the term $3F_{\alpha\beta}(0)$ to $D_{\alpha\beta}$ in (3.6). Now we go over to the system of the φ -functions by the transformation

$$\Phi_{AB}(u) = e^{\frac{1}{2} u_\alpha F_{\alpha\beta} u_\beta} \tau_{AB}(u) \quad (3.9)$$

and deduce the functional equation

$$\left\{ D_{\alpha\beta} \frac{\delta}{\delta u_\beta} + V_{\alpha\beta\gamma\delta} []_{\beta\gamma\delta} - (q_0 i u_\alpha + D_{\alpha\beta} F_{\beta i} u_i) \right\} \Phi_{AB}(u) = 0 \quad (3.10)$$

(cf.¹ for definitions). There only appears a new factor on the right hand side.

For spin 1 no physical solutions can be derived, because the corresponding particles have negative norm³. For the deuteron, too, we cannot expect this approximation to give meaningful results⁴. In the case of physical spin 0 particles we derive with $\kappa l = 6.39$ — this value stems from fermion calculations — the following mass values

Particle	S	I	B	exp. value	Eq. (2.1)	Eq. (2.7)
η	0	0	0	0.584	0.92	0.75
π	0	1	0	0.147	0.30	0.12

§ 3. The Model of the Anharmonic Oscillator

We compare our previous considerations with analogous ones in the case of the anharmonic oscillator. Starting from the equation of motion

$$\ddot{q}(t) = -q^3(t) \quad (3.1)$$

with the commutation relation

$$[q(t), \dot{q}(t)] = q_0 i \quad (3.2)$$

we define the time ordered products

$$\tau_{AB}(1 \dots n) = \langle A | T q(t_1) \dots q(t_n) | B \rangle \quad (3.3)$$

with the generating functional

$$\tau_{AB}(u) = \langle A | T \exp[i \int q(t) u(t) dt] | B \rangle. \quad (3.4)$$

³ H. P. DÜRR et al., Nuovo Cimento **38**, 1220 [1965].

⁴ H. P. DÜRR, Z. Naturforschg. **14a**, 441 [1959].

with

$$[\]_{\beta\gamma\delta} = \frac{\delta^3}{\delta u_\beta \delta u_\gamma \delta u_\delta} - 3F_{\beta i} u_i \frac{\delta^2}{\delta u_\beta \delta u_\gamma} + 3F_{\beta i} F_{\gamma j} u_i u_j - F_{\beta i} F_{\gamma j} F_{\delta l} u_i u_j u_l. \quad (3.11)$$

With the GREEN-function $G_{\alpha\beta}$ for (3.6) it follows after integration

$$\left\{ \frac{\delta}{\delta u_0} + G_{0\alpha} V_{\alpha\beta\gamma\delta} [\]_{\beta\gamma\delta} - (i\varrho_0 G_{0\alpha} + F_{0\alpha}) u_\alpha \right\} \Phi_{AB}(u) = 0. \quad (3.12)$$

(3.12) can be cast into the form analogous to (1.15):

$$\begin{aligned} & \left[\delta_{0\gamma} - 3G_{0\alpha} V_{\alpha\beta\gamma\delta} F_{\delta l} u_l \frac{\delta}{\delta u_\beta} \right] \left[\frac{\delta}{\delta u_\gamma} - F_{\gamma i} u_i \right] \Phi_{AB}(u) \\ &= \left[-G_{0\alpha} V_{\alpha\beta\gamma\delta} \left(\frac{\delta^3}{\delta u_\beta \delta u_\gamma \delta u_\delta} + F_{\beta i} F_{\gamma j} F_{\delta l} u_i u_j u_l \right) + i\varrho_0 G_{0\alpha} u_\alpha \right] \Phi_{AB}(u). \end{aligned} \quad (3.13)$$

The ϱ_0 -term on the right hand side, caused by the canonical quantization, implies an important difference between (1.15) and (3.13).

From (3.12) we get the system of equations for the φ -functions

$$\varphi^{(k+1)}(0 \ 1 \dots k) = G_{0\alpha} V_{\alpha\beta\gamma\delta} [(\beta \gamma \delta \ 1 \dots k)] - \sum_{i=1}^k (i\varrho_0 G_{0i} + F_{0i}) \varphi^{(k-1)}(1 \dots, i-1, i+1, \dots k) \quad (3.14)$$

with

$$\begin{aligned} [(\beta \gamma \delta \ 1 \dots k)] &= \varphi^{(k+3)}(\beta \gamma \delta \ 1 \dots k) + 3 \sum_{i=1}^k F_{\beta i} \varphi^{(k+1)}(\gamma \delta \ 1 \dots, i-1, i+1, \dots k) \\ &+ 6 \sum_{i < j} F_{\beta i} F_{\gamma j} \varphi^{(k-1)}(\delta \ 1 \dots, i-1, i+1, \dots j-1, j+1, \dots k) \\ &+ 6 \sum_{i < j < l} F_{\beta i} F_{\gamma j} F_{\delta l} \varphi^{(k-3)}(1 \dots, i-1, i+1, \dots, j-1, j+1, \dots, l-1, l+1, \dots k). \end{aligned} \quad (3.15)$$

Commuting the coordinates we get k equations for each $\varphi^{(k)}$. We introduce the GREEN-function of (3.6)

$$G(t) = -\frac{1}{2\pi} \int \frac{d\omega}{\omega^2 - 3F(0) + i\varepsilon} e^{-i\omega t}. \quad (3.16)$$

The contraction function F can be given in the general form

$$F(t) = \frac{1}{2\pi} \int_0^\infty \varrho(\omega^2) d\omega^2 \int d\omega' \frac{i}{\omega'^2 - \omega^2 + i\varepsilon} e^{-i\omega' t} \quad (3.17)$$

with $\varrho(\omega^2)$ the spectral density function.

Similar to the spinor case we approximate the spectral function by a single discrete average frequency ω_1 , which roughly should be equal to the frequency of the lowest energy level

$$\varrho(\omega^2) = \delta(\omega^2 - \omega_1^2). \quad (3.18)$$

The numerical value of ω_1 we fix by solving the system of the φ -functions (3.14) with an odd number of legs in the lowest approximation, which yields

$$F(0) = \frac{1}{2\omega_1} = \frac{1}{3} \omega_1^2; \quad \omega_1 = 1.14. \quad (3.19)$$

One obtains by numerical integration of the SCHRÖDINGER equation for the lowest energy level $\omega_1 = 1.09$.

Now we introduce a graphical representation in order to make the following equations more transparent. We shall make use of the FOURIER-transformed equations and therefore have to insert the following explicit expressions:

$$\begin{aligned} \text{---}\overset{\circ}{\omega}\text{---} &= F(\omega) = i/(\omega^2 - \omega_1^2 + i\varepsilon) \\ \text{---}\underset{\omega}{\omega}\text{---} &= G(\omega) = -1/(\omega^2 - 3F(0) + i\varepsilon) = (iF(\omega) \text{ with (3.19)}) \\ \text{---}\cdot\text{---} &\text{gives a factor } (-1); \text{ the sum of all } \omega\text{'s must vanish at each vertex} \end{aligned} \quad (3.20)$$

$$\begin{array}{c} \omega + \omega_2/2 \\ \hline \omega - \omega_2/2 \end{array} \bigg| \omega_2 = \varphi(\omega) .$$

There is to be integrated over all internal variables and multiplied by a factor $1/2\pi$ for every integration.

From (3.14) we get for $K=1$

$$\bigg| \omega_2 = \bigg| \omega_2 + 3 \bigg| \omega_2 \quad (3.21)$$

and for $K=2$ by iterating the uppermost leg

$$\begin{aligned} & \left\{ 1 - 3 \left[\begin{array}{c} \diagup \\ \diagdown \end{array} + \begin{array}{c} \diagdown \\ \diagup \end{array} + \begin{array}{c} \diagup \\ \diagdown \end{array} \right] \right\} \left\{ \begin{array}{c} \diagup \\ \diagdown \end{array} + \begin{array}{c} \diagdown \\ \diagup \end{array} + \begin{array}{c} \diagup \\ \diagdown \end{array} + \begin{array}{c} \diagdown \\ \diagup \end{array} \right\} \\ & = \begin{array}{c} \diagup \\ \diagdown \end{array} - i g_0 \left[\begin{array}{c} \diagup \\ \diagdown \end{array} + \begin{array}{c} \diagdown \\ \diagup \end{array} + \begin{array}{c} \diagup \\ \diagdown \end{array} \right] \begin{array}{c} \diagup \\ \diagdown \end{array} \end{aligned} \quad (3.22a)$$

and by iterating the lowest leg

$$\begin{aligned} & \left\{ 1 - 3 \left[\begin{array}{c} \diagup \\ \diagdown \end{array} + \begin{array}{c} \diagdown \\ \diagup \end{array} + \begin{array}{c} \diagup \\ \diagdown \end{array} \right] \right\} \left\{ \begin{array}{c} \diagup \\ \diagdown \end{array} + \begin{array}{c} \diagdown \\ \diagup \end{array} + \begin{array}{c} \diagup \\ \diagdown \end{array} + \begin{array}{c} \diagdown \\ \diagup \end{array} \right\} \\ & = \begin{array}{c} \diagup \\ \diagdown \end{array} - i g_0 \left[\begin{array}{c} \diagup \\ \diagdown \end{array} + \begin{array}{c} \diagdown \\ \diagup \end{array} + \begin{array}{c} \diagup \\ \diagdown \end{array} \right] \begin{array}{c} \diagup \\ \diagdown \end{array} \end{aligned} \quad (3.22b)$$

We solve (3.22) in the same way as (2.3) and take only the first term in the expansion of the right hand side. Then we insert the result for the 4-point function into (3.21) and obtain respectively

$$\begin{aligned} \bigg| \omega_2 &= 2 \begin{array}{c} \diagup \\ \diagdown \end{array} - 2 \begin{array}{c} \diagdown \\ \diagup \end{array} \\ &- i g_0 \left(2 \begin{array}{c} \diagup \\ \diagdown \end{array} + \begin{array}{c} \diagdown \\ \diagup \end{array} \right) \end{aligned} \quad (3.23a)$$

$$\bigg| \omega_2 = - 3 i g_0 \begin{array}{c} \diagup \\ \diagdown \end{array} . \quad (3.23b)$$

Let us now compare the two first approximations of the anharmonic oscillator and the nonlinear spinor theory; unfortunately the q_0 - and the $F(0)$ -terms destroy the complete analogy. With respect to (3.21) and (2.1) we have no difference in the structure of these equations. However, in (2.7) and (3.23) we see the influence of the quantization procedure. We note that the q_0 -term forms the characteristic term just in the symmetrically iterated equation and by suppressing it we do not only

change more or less (such as in (3.23a)) the numerical value of the right hand side in (3.23b), but we destroy even the structure of this equation.

In the nonlinear spinor theory (cf. (2.6)) only the 6-point function is responsible for the structure of the equation in the case of the symmetric iteration and forms the characteristic term. In the further steps of NTD we have also expressions which stem of $G_{0\alpha} V_{\alpha\beta\gamma\delta} F_{\beta i} F_{\gamma j} F_{\delta l} u_i u_j u_l \Phi_{AB}(u)$ in (1.15). Therefore the structure of the right hand side of the equations for the $2n$ -point- φ -function is not determined only by the φ -function with $2n+2$ legs ($n \geq 3$).

Finally we give the result for the calculation of (3.21) and (3.23). With our approximation (3.18) for F and with $q_0=1$ we obtain in all cases the same result

$$\bigg| \omega_2 = 3 \begin{array}{c} \diagup \\ \diagdown \end{array} \quad (3.24)$$

This yields $\omega_2 = 2.79$, whereas the value calculated in the frame work of the SCHRÖDINGER theory is $\omega_2 = 2.54$.